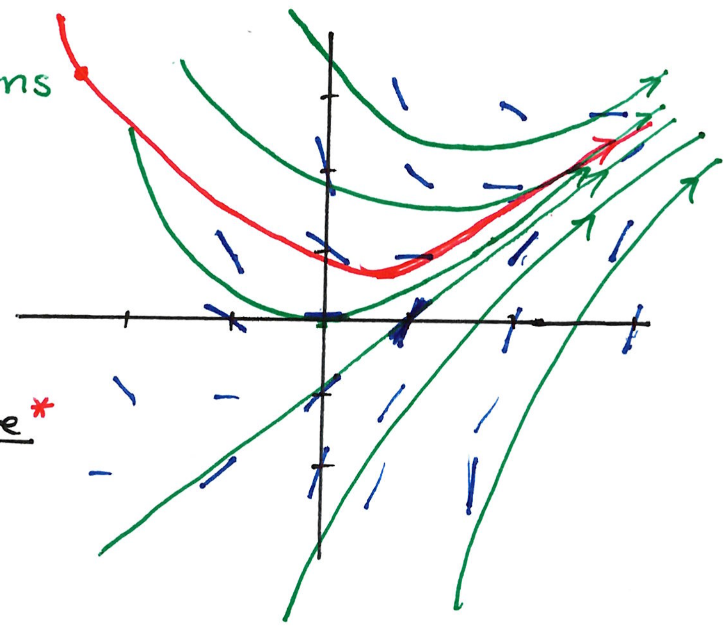
Sec 1.2/1.3 (Blend)

<u>Slope field</u> : an ODE $\{\frac{dy}{dx} = f(x,y)\}$ specifies a slope value (L to R) at each point in (x,y)-plane.

<u>Solution curve</u> ("solution") for ODE $\{y' = f(x,y)\}$ is a curve in x,y-plane having derivatives which "follow the slope field determined by $\{y' = f(x,y)\}$"

<u>Ex</u> Sketch slope field and some solution curves for $\{y' = x - y\}$

Green curves are some solutions (solution curves) for ODE $\{y' = x - y\}$



Ex (cont): Draw a solution curve having <u>initial value</u>* $y(-4) = 3$

* curve must pass through $(-4, 3)$

Red curve is a solution to the <u>initial value problem</u> (IVP) $\{y' = x - y,\ y(-4) = 3\}$

Solutions to ODEs can be:

1) **General**: involves an unknown constant

XOR

Ex: $y = x^2 + C$ gen sol of $\{y' = 2x\}$

$y = \tan(4x^3 + C)$,

2) **Particular**: don't involve unknown constant(s)

(need initial value to make)

(part. sol'ns are solutions to IVPs)

Ex: $y = x^2 + 7$ part. sol. of $\{y' = 2x,\ y(0) = 7\}$

$y = \tan(4x^3 + \pi/4)$ part. sol. of

$\{y' = 12x^2(y^2+1),\ y(0) = 1\}$

A) **Explicit**: solution is written as $y = f(x)$ with some $f(x)$

XOR

$y = x^2 + C$, (expl., general)

$y = x^2 + 7$ (expl., particular)

B) **Implicit**: has the form of an equation (like a level set)

$F(x,y) = C$ or $F(x,y) = (\#)$

Ex: $x^2 + y^2 = C$ or $x^2 + y^2 = 9$

(if not specified, default to finding explicit)

Notes: Not always possible to change implicit sol'n into an explicit one. ($x^2 + y^2 = C \longrightarrow y = \pm\sqrt{C - x^2}$)

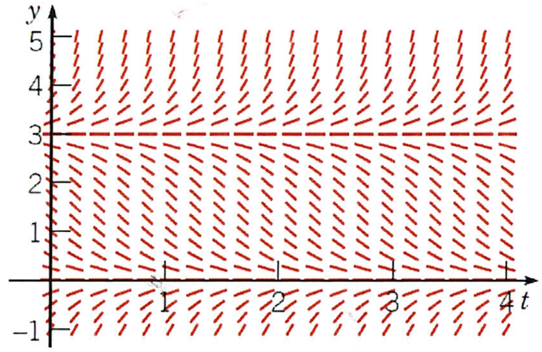
With an implicit solution, it's easy to find ODE/slope-field that "made it": implicit diff.

$x^2 + y^2 = C$

$\longrightarrow 2x + 2yy' = 0$

$\longrightarrow \boxed{y' = -x/y}$

Among the following choices, which ODE has a slope field similar to the one to the right?



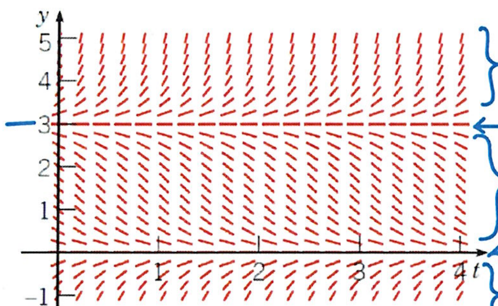
A. $y' = y(3-y)$

B. $y' = y - 3$

C. $y' = y(3+y)$

D. $y' = y(y-3)$

E. $y' = y + 3$



} increasing ($y'>0$) when $y>3$

flat ($y'=0$) when $y=3$

} decreasing when $0<y<3$

$y'=0$ when $y=0$

} $y'>0$ when $y<0$

Thus our description of the function is

$$\begin{array}{lccccc} y': & + & \vdots & - & \vdots & + \\ y: & \longleftarrow & 0 & \text{———} & 3 & \longrightarrow \end{array}.$$

Only choices A and D have $y'=0$ when $y=0$ <u>and</u> $y=3$.

<u>Choice A</u>: $y'=y(3-y)$
$$\begin{cases} \text{if } y<0, & y'=(-)(+)=- \quad \times \\ 0<y<3, & y'=(+)(+)=- \quad \times \\ y>3, & y'=(+)(-)=+ \end{cases}$$

<u>Choice D</u> $y'=y(y-3)$ $\rightarrow$
$$\begin{cases} y<0 \Rightarrow & y'=(-)(-)=+ \checkmark \\ 0<y<3 \Rightarrow & y'=(+)(-)=- \checkmark \\ y>3 \Rightarrow & y'=(+)(+)=+ \checkmark \end{cases}$$

You bail out of a helicopter and pull the ripcord of your parachute. Now the air resistance proportionality constant is k = 1.6, so your downward velocity satisfies the initial value problem below, where v is measured in ft/s and t in seconds. In order to investigate your chances of survival, construct a slope field for this differential equation and sketch the appropriate solution curve. What will your limiting velocity be? Will a strategically located haystack do any good? How long will it take you to reach 95% of your limiting velocity.

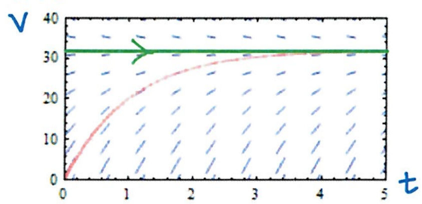
$$\frac{dv}{dt} = 32 - 1.6v,\ v(0) = 0$$

$\uparrow$ air resistance $\propto$ (−vel.)

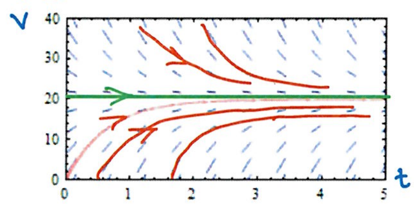
$g \downarrow = 32 ft/s^2$

Construct a slope field for this differential equation and sketch the appropriate solution curve. Choose the correct graph below.

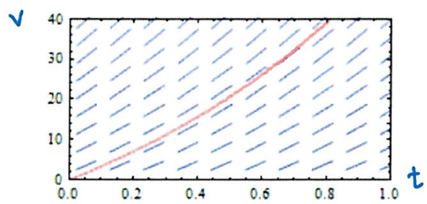
A.



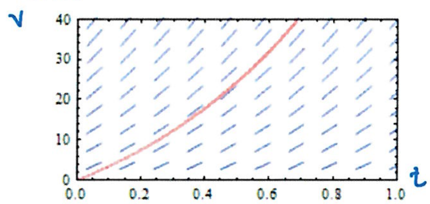
B.



C.



D.



$a(t) = g - kv$ ("down" is "+")

$v'(t) = 32 - 1.6v = 32 - \frac{16}{10}v = 16\left(2 - \frac{1}{10}v\right) = \frac{16}{10}(20 - v)$

$\frac{dv}{dt} = \frac{8}{5}(20 - v)$ (careful that this isn't $\frac{dv}{dt} = f(t)$)

$$\begin{cases} v < 20 \Rightarrow v' > 0 & \text{(speed inc.)} \\ v = 20 \Rightarrow v' = 0 & \text{(no change)} \\ v > 0 \Rightarrow v' < 0 \end{cases}$$

Thus, $v(t)$ tends toward $v = 20$ (terminal velocity)

This is 20 ft/sec ≈ 13.6 mph... you'll live